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(A journal for students)

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R. ATHMARAMAN

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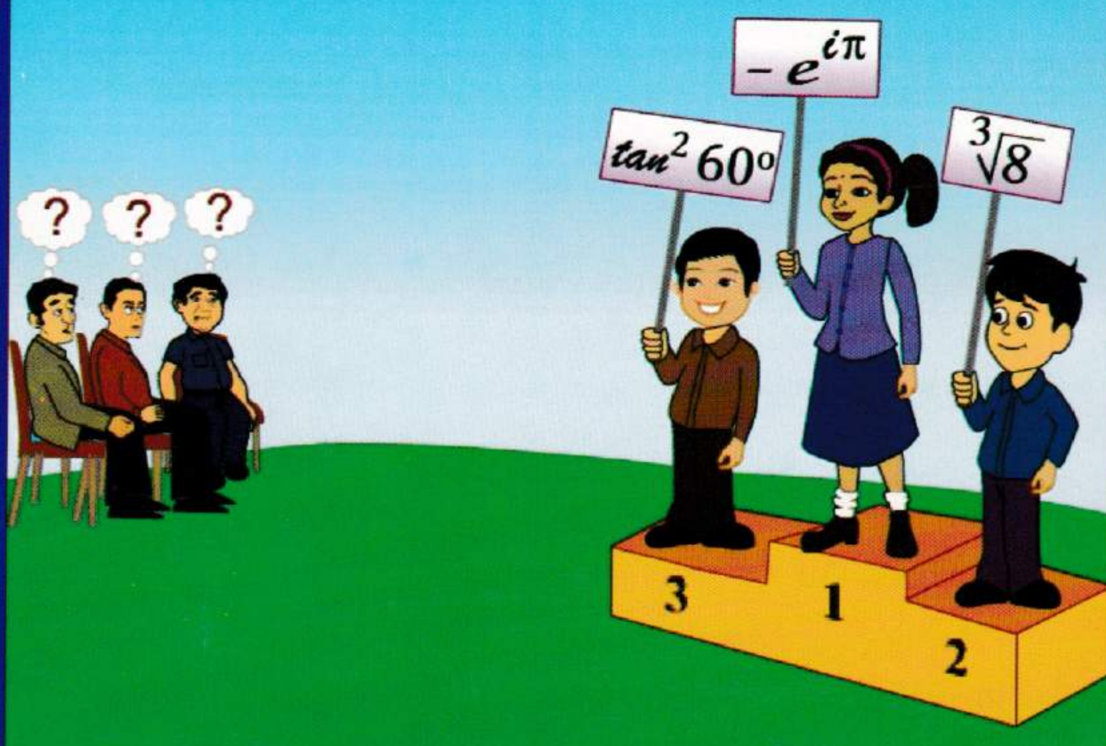
MATHELETICS MEET



JM

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Junior mathematician (JM) is a Mathematics magazine, principally meant for youngsters of age between 8 and 16.

- Aims to interact directly with the fresh, young and receptive minds, motivating them in their appreciation and application of Mathematics.
- Aspires to present Mathematics as a lovable subject, satisfying to the serious-minded and pleasurable for others, removing math-phobia.
- Provokes young student-authors to write their own discoveries and creative thoughts.

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From the desk of the Editor

ON "TAXICAB' NUMBERS"

Sometimes inadvertent omissions in presentations also lead to quite useful discussions. 'Mathemajician' Mr S. A. Rahim refers to page 3 of JM 24 / 3 April 2015 to say that placing 87539319 as being *"next"* to Ramanujan Number 1729 is unfair. In what manner the given number, he asks, is the sum of cubes in two different ways and says that the 'next' number should have been "1404" $= 2^3 + 16^3 = 9^3 + 15^3$. (Here was a small typographical slip, because he would have actually meant the number to be 4104). An infinite such numbers can be generated by the formula: $x^y\ (1^3 + 12^3 = 9^3 + 10^3)$ where x and y are natural numbers and y is a multiple of 3.

The comment demonstrates that the statement in question was somewhat imprecise and wanting in details. The word "next" was not fully articulated and has led to this anomaly. However, the error pointed out by the distinguished reader stimulated us to spell out what we really tried to mean by saying that 87539319 is "next" to 1729, in the pattern discussed.

Recall the famous anecdote of G. H. Hardy's visit to the hospital where Srinivasa Ramanujan was being treated, when the Indian genius pointed out that 1729 is the smallest number expressible as the sum of two cubes in two different ways: 1^3+12^3 and 9^3+10^3. Some say that Hardy then asked for such a number with the sum of two fourth powers in two different ways but Ramanujan did not happen to know. Hardy could have easily asked for a similar number which is a sum of two cubes in three (or even more) distinct ways.

We may name, as many mathematicians do,the lowest number that can be expressed as the sum of two positive cubes in n different ways as a "taxicab number".

1729 is called Taxicab(*2*) because n is 2.

87539319 is Taxicab(*3*), comes *next.* It is the lowest number that can be written as the sum of two positive cubes in three different ways:

Taxicab(*3*) $= 87539319 = 167^3 + 436^3 = 228^3 + 423^3 = 255^3 + 414^3$

[However, if you are allowed to use both positive and negative integers, there is the much smaller solution, namely $4104 = 2^3 + 16^3 = 9^3 + 15^3 = (-12)^3 + 18^3$]. More interesting information on this topic, is available in the article available on the web: "Taxicabs and Sums of Two cubes" by Joseph H. Silverman.

Thanks to Mr. Rahim (mathmaj2013@gmail.com) for initiating the discussion.

ONE, TWO, THREE, ...,

One more contribution from Mr Rahim.

Thousands of solutions have been found for 1 to 9 to Give 100, using all the digits, 1 to 9, in that order and each only once. The symbols $+$, $-$, $\times$, $\div$, $\sqrt{}$, Σ, and groupings, powers, decimals and factorials – totally 10 operations are used.

For example :

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9) = 100$$

$$123 - 45 - 67 + 89 = 100$$

$$\{\cdot 12 \div (\cdot 3 \times \cdot 4)\}^{5! - \sqrt{6}}\}\{\Sigma\, 7 + (8 \times 9)\} = 100$$

However, only 15 solutions were available using only + and ×. They are

i. $1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9)$

ii. $[(1 + 2 + 3) \times 4] + 5 + 6 + (7 \times 8) + 9$

iii. $[(1 + 2) \times 3] + (4 \times 5) + 6 + (7 \times 8) + 9$

iv. $[(1 + 2) \times 3 \times 4] + 5 + (6 \times 7) + 8 + 9$

v. $\{1 \times [2 + (3 \times 4)] \times 5\} + 6 + 7 + 8 + 9$

vi. $1 \times [2 + \{3 \times (4 + 5)\} + 6 + (7 \times 8) + 9]$

vii. $(1 \times 2) + [3 \times (4 + 5)] + 6 + (7 \times 8) + 9$

viii. $[\{(1 \times 2) + (3 \times 4)\} \times 5] + 6 + 7 + 8 + 9$

ix. $[1 \times 2 \times (3 + 4) \times 5] + 6 + 7 + 8 + 9$

x. $1 \times [2 \times (3 + 4) \times 5] + 6 + 7 + 8 + 9$

xi. $1 \times \{[2 \times (3 + 4) \times 5] + 6 + 7 + 8 + 9\}$

xii. $(1 \times 2 \times 3) + 4 + 5 + 6 + 7 + (8 \times 9)$

xiii. $1 \times [(2 \times 3) + 4 + 5 + 6 + 7 + (8 \times 9)]$

xiv. $(1 \times 2 \times 3 \times 4) + 5 + 6 + (7 \times 8) + 9$

xv. $1 \times [(2 \times 3 \times 4) + 5 + 6 + (7 \times 8) + 9]$

It gets difficult to find solutions using a few combinations of symbols, such as given below; there may exist some. JMs interested in this Game may pursue further.

(1) ***+, −, ÷, Powers***

(2) ***−, ×, ÷, √, Grouping***

(3) ***+, □ , !***

(4) ***+, −, □ , □***

(5) ***+, □ , □ , (.)***

A new learning experience:

AN IN-HOUSE CAMP FOR MATH TEAM

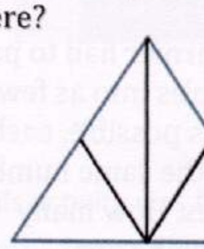
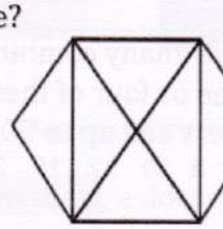
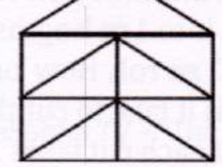
(An academic exercise as narrated by Math Dept., T.I. School, Chennai)

Here is a fascinating report about an 'in-house' Math Camp, conducted for the students of TI School (Ambattur) and AMM School (Kotturpuram). Under this scheme, students and teachers stay in a place for a couple of days and indulge in leisurely (and at the same time enlightening) activities related to Mathematics. The camp reported about was on 14th and 15th of August 2015. After initial formalities, the groups were divided into ten groups, which were named after various topics of Mathematics, like Trigonometry, Probability, etc.

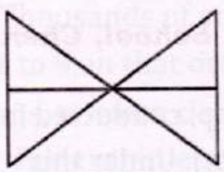
The first pair of activities was Puzzles and Origami; five of the groups opted for Puzzle experience while the remaining favoured to venture with Origami.

'Math and Puzzles'

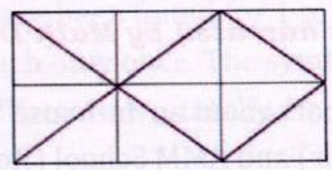
The Puzzles were simple, yet challenging. Here follow some samples from the set of questions used. (Answers are given elsewhere in this journal):

1. How many triangles are there? 	2. How many triangles are there? 	3. How many triangles are there? 

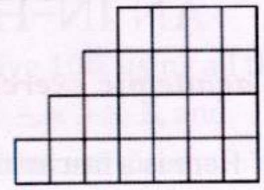
4. How many triangles are there?



5. How many triangles are there?



6. How many squares are there?



7.
The Barber of Seville shaves all the men living in Seville. No man living in Seville is allowed by law to shave himself. The barber of Seville lives in Seville. Who shaves the Barber of Seville?

8.
Arrange these shapes in order according to the number of sides, starting with the one with the least number:

octagon ,
hexagon,
pentagon and
heptagon

9. What will be the result if the hands of this clock are moved as follows:

A,forward hrs, 15 min.
B. back 4 hrs, 25 min.
C. back 1 hr, 30 min.

10.
A. What starts the top series?
B. What ends the bottom series?

?8 1 6 3 2 6 4 1 2 8
1 9 3 8 7 6 1 5 2 3 0 ?

11.
In a party of 35 people there are twice as many women as children and twice as many children as men. How many of each are there?

12.
If I had one more sister I would have twice as many sisters as brothers. If I had one more brother I would have the same number of each. How many brothers and sisters have I ?

13. Which is the odd number out?

3 11 17
7 15 29

14. What comes next?
208 CIV 52XXVI ?

15. Put arithmetic signs between given numbers to make the equation true:
18 2 9 24 5=100

16. A machine cuts rolls of cloth into 1 ***m*** lengths, from a 200 ***m*** roll. How long would it take to cut the full roll if each cut takes 1 sec?

17. How many combinations of three or four of these numbers add up to 50?
2 4 6 8 10 17 19 21 25

18. A farmer had to pack 441 apples into as few boxes as possible, each having the same number of apples. How many boxes did he use?

19. .7 = ? %	20. What is the sum of the integers from -3 to 3?	21. Among the numerals 0 to 99 which is the least repeated digit?
22. The only number following a square and preceding a cube that is smaller than 50 is ____.	23. Product of 9 negative integers and 10 positive integers is a _____ integer.	24. In which town was Srinivasa Ramanujan born?

To secure maximum points, the groups vied with each other and tried their best to complete as many puzzles as possible.

The group that transacted with 'Math and Origami' consisted mostly of students of class twelve and still this was a mesmerizing experience for them. They were supported by inputs from some eight volunteers and a few teachers.

The work out began with making a 'tetrahedron', a three-dimensional figure made of 4 triangles. There was discussion on examples (like a pyramid) and counter examples for a tetrahedron. A tetrahedron is a triangular pyramid. "Do we have pyramids that are not tetrahedrons?" - was a typical query leading some discussion. Several aspects of Origami were taken up one after another. The results were amazing! They were judged on the basis of timing, perfection and team work.

Math and Origami'

This activity was followed by a Video show, a documentary by RajyaSabha TV –"The maths factor-The magic of Algebra". Starting with the mathematics the game of Chess in India, it described the travel of algebra to Greece, Persia and then to the whole world.

After the video show, the feedback obtained from the students was quite encouraging. It enabled an informal discussion on the History of mathematics and how it is a universal language. (For information: Most of these videos are available on YouTube also). Though the video was only a general introduction on the emergence and utility of Algebra, the content and followed the contents

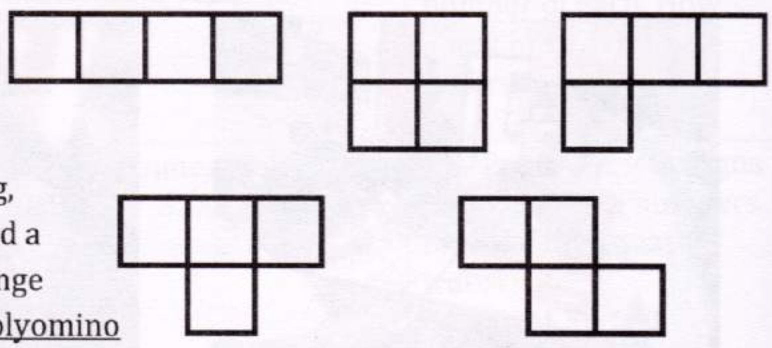
The Maths Factor

Wednesday 9am on DD

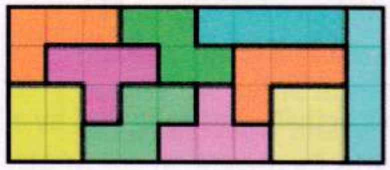
The post-dinner session was replete with games, brainteasers and challenges on critical thinking. There were 'Challenging with tetraminoes', 'Match stick puzzles', 'Tower of Brahma', 'Scavenger Hunt', 'Back to Back mystery', 'Memory Orientation', 'Hopping', 'Punching Patterns', 'Identifying the odd one out' and similar conundrums. Collaborative effort of the participants became apparent during the session' varied assignments.

Tetraminoes

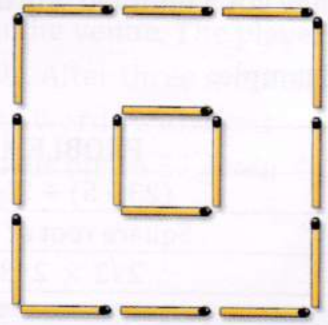
A **polyomino** is a plane geometric figure formed by joining one or more equal squares edge to edge. Translating, rotating, reflecting, or glide reflecting what is called a "free polyomino" does not change its shape A tetromino is a 4 polyomino There are five free tetrominoes. These ar called "straight", "square", "L-tetromino", "T-tetromino", and "Z-tetromino".



One sample game is: Use tetrominoes to produce a rectangle (with no over-lapping) which contained quotes relating to behavior, attitude etc. Amazingly, almost all the teams got full points in this exercise!



Match Stick Puzzles



Matchstick puzzles are rearrangement **puzzles** in which a number of **match sticks** are arranged as some geometric figure. The problem usually looks like this: "move ***n*** **matchsticks** to make ***m*** squares (or triangles, or rectangles, or circles)".This hands-on mathematics was very much enjoyed by the children of middle classes. ***Here is a try-out***: From the given figure move 4 sticks to make 3 squares!

Tower of Brahma

The **Tower of Brahma** or **Lucas' Tower**, is a puzzle with a number of disks of different sizes which can slide onto any rod. The puzzle starts with a conical arrangement of disks in a neat stack in ascending order of size on one rod, the smallest at the top. The student is expected to move the entire stack to another rod, obeying the following simple rules:

1. Only one disk can be moved at a time.
2. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack i.e. a disk can only be moved if it is the uppermost disk on a stack.
3. No disk may be placed on top of a smaller disk.

With three disks, the puzzle can be solved in seven moves. The minimum number of moves required to solve is $2^n - 1$, where n is the number of disks.

Scavenger hunt

Scavenger hunt involves solving with given clues to arrive at the final answer. It involved minimum mathematical skills and more of a generalvariety. Each team was made to face a series of clue-giving questions. Answering each question, the

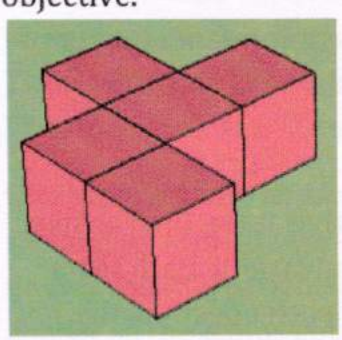
teams were to arrive at a number ranging from 1 – 26; and decoding this with A – Z they got a jumbled word to be rearranged to get the final answer.

Example:

PROBLEM	ANSWER	LETTER	WORD
$(23 - 5) \div 3 - 4$			
Square root of 529			
$2\sqrt{3} \times 2\sqrt{3}$			
$9 - 2 + 4 - 1 + 5$			
$(80 - 20) \div 4 - 2 + 7$			
Eighth Prime			
$(17 - 2) \div 5 + 2$			
$(7 + 3 - 1 + 2 - 3) \times 2$			
Sum of first two prime numbers			

'Back to Back'

Yet another exciting time-boundgame was based on the properties of two dimensional and three dimensional figures. In this game, two students sit *back to back*. One student creates a 3-dimensional design using pattern blocks or other sets of shapes. The student then describes the design to the other student who tries to build an exact copy. The students switch the roles during the next round. In the course of the game the students become conversant in using Positional Vocabulary, Shape Vocabulary etc. and become more comfortable with geometric terminology. A slightly modified procedure was followed, keeping intact the main objective.



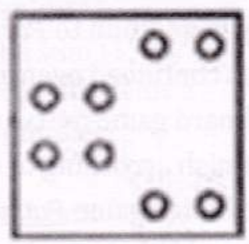
Memory Game

Words related to Math were stuck at different places in the venue. The players were given three minutes to go round and search for the words. After three minutes the team members had to assemble in a place and list down the words with right spelling. Out of 10 teams one team found a maximum of 27 words out of 37 given words.

Hopping Game

This game was played simultaneously by two teams. Each team stood in a line behind the starting crease. Two tables were provided for each team. The first table had cubes and cuboids of various sizes. One by one the students would get a question. In case the person was not able to answer, he/she had the liberty to go back to their team and discuss. If they answered right, they got two points and had to hop and collect a cube or cuboid from the first table and place it on the second table. The objective of the game is to build the highest building. Ten points were awarded to the highest building. Quick thinking and mental calculation were the important factors of this game.

Punching Patterns



The students were provided with a square sheet of paper in which a pattern of dots was provided. The students had to fold a plain white square sheet given to them, make holes in it using the punching machines and obtain an exact duplication of the pattern provided to them. (An example is shown here).

Finding the odd one out

The students were asked to identify the odd one out of the given collection of numbers. The collection is given in such a way that each one except one satisfies a particular definite property. The one which does not satisfy that characteristic is to be taken out.

396, 462, 572, 427, 671, 264

(Note: In each number except 427, the middle digit is the sum of other two).

The first day of the Fun ended with a 'camp fire'.

Maths around us:

The next day was a special day; Our Independence Day. A group of students went crazy about Rangoli sketches they were asked to design and execute; there were

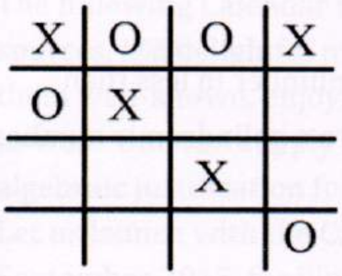
beautiful presentations exhibiting different types of symmetries providing learning experience to participants. The other group of participants went for a "Nature walk" to appreciate symmetry present in and around the campus, listing them quite systematically.

Traditional games:

In the morning, after physical exercises illustrating some mathematical concepts, the members were introduced to various traditional games. *Pallankuzhi* was the pick of the day in addition to *Paramapadam.*

This continued even after the Independence Day celebrations. Chathurang (Chess), Ludo [a board game for two to four players, in which the players race their four tokens from start to finish according to die rolls. Like other cross and circle games, Ludo is derived from the Indian game Pachisi, but simpler], Tic-tac-toe (a familiar game for two players, called "X" and "O", who take turns marking the spaces in a 3×3 grid.

The player who succeeds in placing three respective marks in a horizontal, vertical, or diagonal row wins the game), and *AduPuliAttam*were included. All the participants could explore little mathematical tricks that play a significant role in these games.



× and Ö

Ludo

Chess

Investigations:

The next session was reserved for little mathematical investigations. An outline of 'how to go about' was given and each group was provided with a work sheet comprising of a proposal for investigation followed by some hints. It was emphasized that the points will be based on the 'processes' than on 'products'. Students had to think out of box and investigate.

Here is a sample:

A **palindromic number** reads the same both forward and backward. For example, 52825 is a palindromic number. Try to list a few palindrome numbers.

In what follows, the a number (75) is added to its reverse (57), then the sum (132) is added to its reverse (231), and so forth. Repeating this process four times yields a palindromic number. Will this process always result in a palindromic number?

75	255
+ 57	+ 552
132	807
+ 231	+708
363 (Palindrome!)	1515
	+5151
	6666 (Palindrome!)

1. Will this always result in a palindromic number?
2. Find one or more two-digit numbers for which this process requires 2 steps, 3 steps, 4 steps, 6 steps, and 24 steps.
3. Find 3 numbers <100 that require at least 4 additions to result in a Palindrome.
4. Is there a relationship for two-digit numbers between the number of steps to get a palindromic number and the sum of thedigits of the original number?

5. Try some three-digit numbers and look for patterns.

6. There are only 13 three-digit numbers that do not lead to a palindromic number in 23 or fewer steps. Find the first few of these numbers and look for patterns in the numbers and the sums of their digits.

7. Over 97% of all four-digit numbers lead to a palindromic number in less than 22 steps. Find the first few of the numbers that do not lead to a palindromic number in less than 22 steps.

Math and frieze patterns :

The final piece of the in-house programme was on patterns generated by various types of transformations like Translation, Rotation, Reflection, Glide reflection and similarities. A video programme was shown touching on these concepts and the patterns produced based on them.

Then the groups were left to create their own frieze patterns; they had to make patterns of aesthetic value and explain the types of techniques used to create them. The best among them were given special credits.

The programme came to an end after distributing prizes for the top scorers and participation certificates for all the students. Students left the campus with new ideas, new friends and a new outlook on mathematics.

Note: ***The detailed account of the camp given here is to motivate JMs to think of uch activities in their own institutions. Learning math ideas does not end with experiences in the classroom. Interaction, Innovation and Open-ended Investigation can make mathematics lively, enjoyable and purposeful.***

FUN WITH CALENDAR

T. Dharmarajan, Math Education Specialist, Teachers' Colony, Coimbatore 641022.

The following Calendar tricks, compiled from various sources, is a delightful mine of surprises, some of them well-known, enjoyable by all, in the classrooms as well. One can supply exceptionally simple algebraic justification for the tricks.
Let us launch with the Calendar for the month of September 2015, for illustration.

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

Trick 1:

Take any 2×2 square as shown here.

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

i) You can tell the dates when the total is given.
In the case here, the total = 15+16+22+23 = 76.

If this total is known, you can very easily see that

The first number of the square = $\frac{76}{4} - 4 = 15$.

From this the other numbers can be found. $\frac{76}{4} - 4 = 15$

Reason: If you assume the first number to be x, then the square gives the total ***4x+16=4(x+4).***

x	$x+1$
$x+7$	$x+8$

ii) ***You can tell the total when the smallest number in the square is given. In this case, the total = 4(15+4) = 76. [Thus, if the*** smallest number is x, then the total = ***4(x+4)].***

Trick 2:

Suppose you take a 3× 3 square as shown here, then it is of the form

y-8	***y-7***	***y-6***
y-1	***y***	***y+1***
y+6	***y+7***	***y+8***

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

which gives the total 9***y***.

i) ***You can tell the dates when the total is given.***

In this case, if the total is given as 144 (by adding 8, 9, 10, 15, 16, 17, 22, 23 and 24), then $9y = 144$ gives the middlemost number y to be 16; other dates are now easily found.

ii) ***You can tell the total when the smallest number in the square is given.***

Let us illustrate the process. Here the smallest number is 8. This means $\boldsymbol{y}$- $8 = 8$ and hence $\boldsymbol{y}$= 16 from which you get $9\boldsymbol{y} = 144$.

Extension: Find the total, when the largest number in the square is given.

Trick 3:

Consider any 4×4 square as shown here.

It is of the form

n	*n+1*	*n+2*	*n+3*
n+7	*n+8*	*n+9*	*n+10*
n+14	*n+15*	*n+16*	*n+17*
n+21	*n+22*	*n+23*	*n+24*

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

and the grand total is $16\boldsymbol{n}+ 192$.

i) ***You can tell the dates when the total is given.***

Here the total =6+7+8+9+13+14+15+16+20+21+22+23+2+28+29+30= 288.

So, $16\boldsymbol{n}+ 192. = 288$ gives $\boldsymbol{n}$ =6 which is the smallest number from which other numbers can be obtained.

ii) ***You can tell the total when the smallest number in the square is given.***

After all, we need in our case $16 \times 6 + 192$, which is 288.

Extension: Extend this to the case of a rectangular block (say 3×4) is given.

Trick 4:

How will you find the dates when you are given the total of the dates forming a Cross as shown in the adjoining figure?

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

The general form here is

	$a-7$	
$a-1$	a	$a+1$
	$a+7$	

whose total is $5\boldsymbol{a}$.so to find *a,* just divide the total by 5. In our example the total is 75 and hence the middle number is $75 \div 5 = 15$.

Extension:

If instead of a cross-shaped block,
a H-shpaed pattern is given, then the middle most number is one-seventh of the total.
Do you see 'how'?

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

Trick 5:

We now connect the entries of a calendar sheet to the idea of a Magic Square.
Suppose some 3×3 square block in the calendar sheet is given (as shown here). It is not a magic square but it can be modified into a magic one. Observe the steps shown by the following steps:

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

Step 1:

2	3	4
9	10	11
16	17	18

⇒

...	4	...
...	10	...
...	16	...

Total 30

Step 2:

2	3	4
9	10	11
16	17	18

⇒

...	4	...
18	10	2
...	16	...

Step 3:

2	3	4
9	10	11
16	17	18

⇒

9	4	...
18	10	2
...	16	11

Step 4: The remaining numbers are then easily filled up, to get the magic total 30.

9	4	17
18	10	2
3	16	11

This was an easy case of 3×3 arrangement.
What happens to the case of 4×4 pattern is also interesting.

Here is an example:

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

Step 1:

6	7	8	9
13	14	15	16
20	21	22	23
27	28	29	30

⇒

30	...	...	27
...	...	...	...
...	...	...	...
9	...	...	6

Step 2:

6	7	8	9
13	14	15	16
20	21	22	23
27	28	29	30

⇒

...	...	...	...
...	22	21	...
...	15	14	...
...	...	...	...

Step 3:

6	7	8	9
13	14	15	16
20	21	22	23
27	28	29	30

⇒

30	7	8	27
13	22	21	16
20	15	14	23
9	28	29	6

⇑

This Square is Magic!

Trick 6 :

Here is another type of square, but not magic in its usual sense; it has other concurring totals. Consider any 4×4 square block as shown here.

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

The Red total = $1+4+22+25$ $=52$

The Green total = $8+11+15+18$ $=52$

The Bluetotal = $2+3+23+24$ $=52$

The **Yellow** total = $9+10+16+17$ $=52$

...	2	3	...
...	...	...	...
...	...	...	...
22	...	...	25

$= 2 + 3 + 22 + 25 = 52$

...	...	...	...
8	...	...	11
...	16	17	...
...	...	...	...

1	...	...	4
...	...	...	...
...	...	...	...
...	23	24	...

$= 52$!

Some more ideas:

i) If the top-left number (here 1) is given, can you predict the total of all the numbers in the block? I (always!) added 24 to 1 and then added 1 once more to get 26; I multiplied this 26 by 8. The total 208 was correctly obtained. Can you find why I followed this algorithm?

ii) If the top-right number (here 4) is given, can you predict the total of all the numbers in the block? Try to find an algorithm.

Trick 7 :

Now we consider numbers on the calendar that occupy the position like an isosceles triangle.

Two such patterns are shown here. Theaim is find the total of numbers lying along the triangle, when a top corner (vertex) value is given.

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

(For the green triangle, we consider the lower Vertex).

We have several such totals. Here are some:

$$8 + 16 + 24 + 23 + 22 + 21 + 20 + 14 = 148$$
$$9 + 17 + 25 + 24 + 23 + 22 + 21 + 15 = 156$$
$$10 + 18 + 26 + 25 + 24 + 23 + 22 + 16 = 164$$
$$29 + 21 + 13 + 14 + 15 + 16 + 17 + 28 = 153$$
$$17 + 9 + 1 + 2 + 3 + 4 + 5 + 11 = 52$$

Suppose the top vertex x (say 9) is given, simply use the formula $8x + 84$, to get the total as $8 \times 9 + 84 = 72 + 84 = 156$.

Similarly if 10 is given, the total is $8 \times 10 + 84 = 80 + 84 = 164$.

Can you find why the formula $8x + 84$ is used here?

When the isosceles triangular pattern is vertex downwards, the formula $8x - 84$ is used. It is not difficult to find why it is so.

September 2015

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

Extension:

You can extend this kind of game to other patterns as well.

Here is a trapezium pattern.

If the shorter parallel sides of the trapezium are up, then if 7 is given,

The sum total of the numbers along the figure is now found as 73. Simply use the formula $6x + 31$.

If the trapezium is upside down, the formula is $6x + 23$.

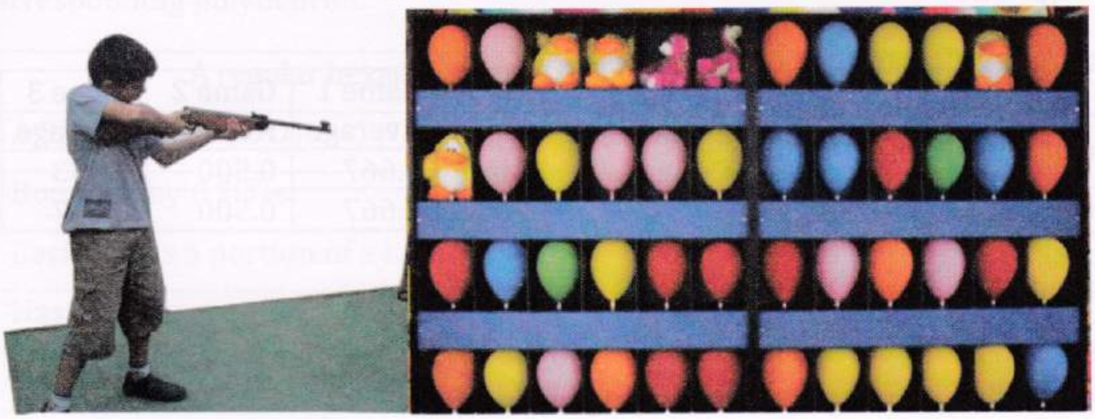
Answers to Puzzles under AN IN-HOUSE CAMP FOR MATH TEAM

1. 7	9. 5.15, 12.50,11.20	17. ....
2. 10	10. 4,4	18. 21
3. 11	11. 5m,10ch,20 wom.	19. 70%
4. 6	12. 3 sisters,2 brothers	20. 0
5. 23 or 25	13. 15	21. 0
6. 40	14. 13	22. 26
7. Woman	15. 18/2 x 9 + 24 – 5= 100	23. Negative
8. Penta,hexa,hep ta,octa	16. 199 sec	24. Erode

UNTIDY APPROACH TO "AVERAGES"

Maithreyaa S, Standard VI,SSM Sr.Sec.School, Chennai- 600 063

A lot of balloons hang at a screen. Balloon shooting game is where you need to shoot as many balloons as possible within a prescribed time-slot. You get as many points as the number of balloons you shot out successfully. Ajay and Stella decide to play the 3 games.



Game 1 begins.

Ajay starts first. He attempts 30 times and succeeds shooting 20 balloons.
Thus the average of his shooting skill is $20 \div 30 \approx 0.667$
Stella is initially nervous about the game. She tries only 3 times and achieves shooting out 2 balloons.
Thus the average of her shooting skill is also $(2 \div 3) \approx 0.667$

	Game 1 Average
Ajay	0.667
Stella	0.667

Game 2 starts then.

It starts raining and so each of them decides to attempt only two times. Surprisingly, both perform identically, securing 1 shoot-out for 2 attempts and thus each of them obtain a skill average 0.500.

	Game 1 Average	Game 2 Average
Ajay	0.667	0.500
Stella	0.667	0.500

Thus both Ajay and Stella have the same average in Game 1 and also in Game 2. Should you say nowat the end of the two games their skill average is same? Let us study their performances.

	Total Number of attempts in Game 1 + Game 2	Total number of Successes achieved	Overall Average
Ajay	$30 + 2 = 32$	$20 + 1 = 21$	$\frac{21}{32} = \mathbf{0.65625}$
Stella	$3 + 2 = 5$	$2 + 1 = 3$	$\frac{3}{5} = \mathbf{0.6}$

Amazing! Their averages are not same now!

There is one more game left. Let us see how it will affect the averages.

Game 3 begins now.

Both Ajay and Stella attempt 12 times. Ajay succeeds 4 times while a confident Stella is triumphant now 8 times. In game 3,

Ajay's average $= \frac{4}{12} \approx 0.333$ and

Stella's average $= \frac{8}{12} \approx 0.667$

	Game 1 Average	Game 2 Average	Game 3 Average
Ajay	0.667	0.500	0.333
Stella	0.667	0.500	0.667

Stella seems to have overtaken Ajay! How will the overall average look, if we take into account all the three games? Let us explore now.

	Total Number of attempts in Game 1 + Game 2 + Game 3	Total number of Successes achieved	Overall Average
Ajay	$30 + 2 + 12 = 44$	$20 + 1 + 4 = 25$	$\frac{25}{44} = \mathbf{0.568}$
Stella	$3 + 2 + 12 = 17$	$2 + 1 + 8 = 11$	$\frac{11}{17} = \mathbf{0.647}$

It must be clear now that the "average" of skills we discussed is not the same in the usual sense. You see that the same average is achieved with different number of hits. This is where the concept of 'weighted average' becomes important. It will be fruitful for JMs if they discuss about such 'mistreatment' of concepts.

Maths in the News!

"An Indian bride walked out of her own wedding ceremony after the groom failed to solve a simple Maths test, according to police in Northern India.

The bride in the state of Uttar Pradesh asked the groom to add 15 + 6. When he gave his answer as 17, she walked out of the ceremony.

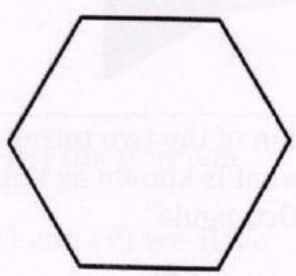
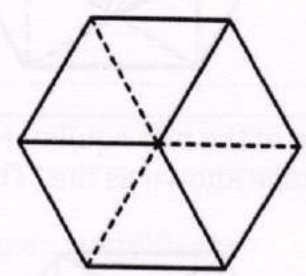
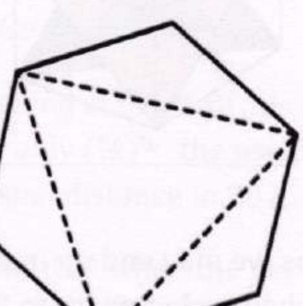
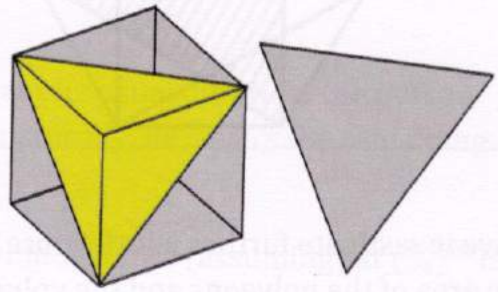
The incident in Rasoolabad village prompted the groom's relatives to plead with her to reconsider. She refused, saying that she had been misled about his level of education."

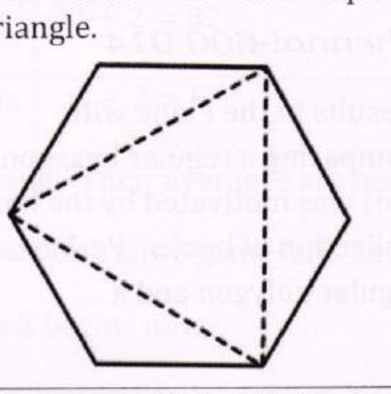
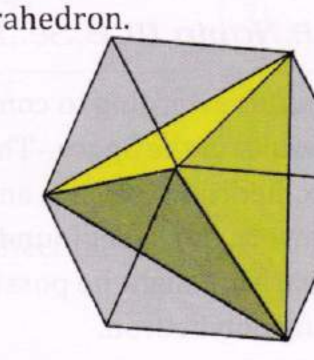
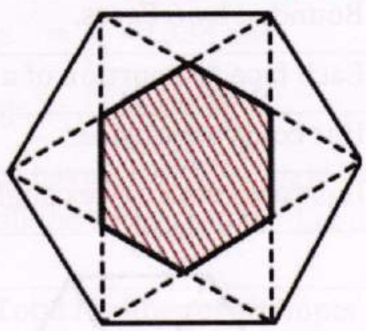
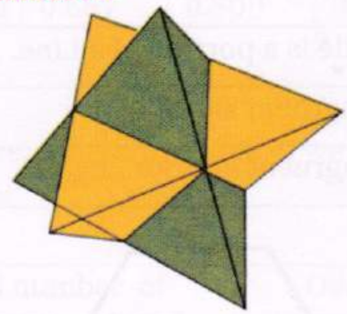
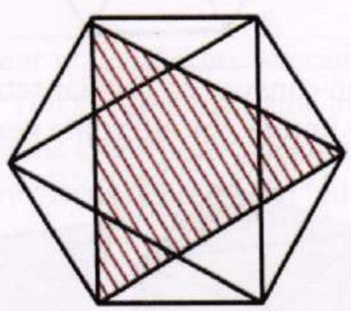
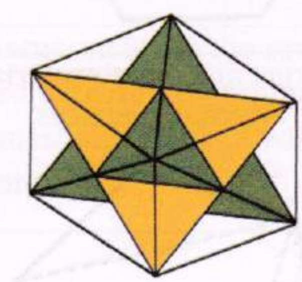
Source: The Times

THE PLANE AND THE SPACE

L.Navin,III B.Sc.,New College, Chennai-600 014

It is quite rewarding to compare geometrical results in the Plane with analogous results in the Space. The following piece comparing a regular hexagon and a regular hexahedron (which is another name for Cube) was motivated by the hazy contents in aworn out sheet found in t he midst of a collection of books. Perhaps similar comparison might be possible between any regular polygon and a corresponding polyhedron.

A regular hexagon	A regular hexahedron
A Plane figure. **(2-D)**	A Solid shape. **(3-D)**
Bounded by 6 **Sides.**	Bounded by 6 **Faces.**
Each side is a portion of a Line.	Each face is a portion of a Plane.
Has congruent sides.	Has **congruent faces.**
Has **congruent interior angles.**	Has **congruent vertex angles**
	
Can construct an **equilateral triangle** by joining the alternate vertices.	Can construct a **regular tetrahedron** by joining the alternate vertices.
	

There is one more such equilateral triangle. 	There is one more such tetrahedron. 
The two equilateral triangles thus formed enclose a **polygon**. This polygon is a **regular hexagon.** 	The two tetrahedra formed thus enclose a **polyhedron.** This polyhedron is a **regular octahedron.** 
The union of the two equilateral triangles form a shape known as the **"The Star of David".** 	The union of the two tetrahedra forms what is known as Kepler's **"Stella Octangula".** 

If we investigate further a little more deeply, perhaps we may end up in comparing the area of the polygons and the volumes of the polyhedra. Let us try to find such comparisons between other analogous figures also.

TWO INTERESTING PROBLEMS

Joel Rajakumar, *Class XI, Hiranandani Upscale School, Egathur, OMR, Chennai*

(This student has taken up a LEAP Course of Ladder Education providers. He got interested in two questions very much and has come out with a couple of solutions for each. He feels rewarded by finding alternate solutions).

Problem 1:

Walking at (¾)th of his usual rate, a man reaches his office 20 minutes late. Find his usual time.

When I first saw this question, I thought of using the idea and forms of *linear equations* since that would fit well easily. ' However I found that there was , as such, no necessity for this process. A simple logic would suffice to arrive at the solution. For comparison, here are both solutions in detail, one by one.

Method 1: Use of Linear Equations.

Let **s** be the usual rate for covering a distance ***d*** in time ***t*** minutes. We get,

$$s = \frac{d}{t} \text{ or } d = st. \quad \ldots\ldots(1)$$

As per the problem, $\frac{3}{4}s = \frac{d}{t+20}$ ......(2)

Solving (1) and (2) we have $t = \frac{3}{4}(t+20)$, which again simplifies to

$t = 60.$

Thus the person usually takes 60 minutes to reach the office.

Method 2: Use of simple logic.

Travelling at (¾)th of his usual rate and at t he usual duration, the person would cover only (¾)th the usual distance. So, naturally, he covers the remaining (¼)th the usual distance in 20 minutes.

Thus it is clear that when the person walks at a uniform rate (assuming) at (¾)th of his usual speed, it takes 4 times 20 minutes or 80 minutes.

We see that this 80 minutes is 20 minutes more than the usual time. Therefore, his usual time is 60 minutes.

Problem 2: If A_1 and A_2 are the AM's and G_1, G_2 are the GM's between two positive real numbers and b, then show that

$$\frac{A_1+A_2}{G_1.G_2} = \frac{a+b}{ab}.$$

This is a very simple elementary problem but the way it yields to 4 different ways of solving it, is quite fascinating.

Method 1: Since a, A_1, A_2, b are in A.P. and a, G_1, G_2, b are in G.P.,
$b = a + 3d$ where d is the common difference of the A.P. and
$b = a.r^3$, where r is the common ratio of the G.P.

We hence get $$d = \frac{1}{3}(b-a)\ ;\ r = \left(\frac{b}{a}\right)^{1/3}$$

Now,
$$\frac{A_1+A_2}{G_1.G_2} = \frac{a+\frac{1}{3}(b-a)+a+\frac{2}{3}(b-a)}{a.\left(\frac{b}{a}\right)^{1/3}.a.\left(\frac{b}{a}\right)^{2/3}}$$

$$= \frac{2a+(b-a).1}{a^2.\left(\frac{b}{a}\right)^1} = \frac{a+b}{ab}$$

Method 2:

$$\frac{A_1+A_2}{G_1.G_2} = \frac{a+d\ +\ a+2d}{ar.ar^2} = \frac{a+\ a+3d}{a.\ ar^3} = \frac{a+b}{ab}$$

Method 3:
i) In an A.P., the sum of the equidistant terms from the extremes is a constant term.
ii) In a G.P., the product of the equidistant terms from the extremes is a constant term.

Therefore, $$a+b = A_1 + A_2 \quad \text{and} \quad a.b = G_1.G_2$$

This straight away gives, $$\frac{A_1+A_2}{G_1.G_2} = \frac{a+b}{ab} \quad \frac{A_1+A_2}{G_1.G_2} = \frac{a+b}{ab}$$

Method 4:
Since a, A_1, A_2, b are in A.P. we may take $A_1 = a + d;\ A_2 = b - d$.

Since a, G_1, G_2, b are in G.P, we may take $G_1 = a.r\ ;\ G_2 = \frac{b}{r}$

$$\frac{A_1+A_2}{G_1.G_2} = \frac{a+d+b-d}{a.r\frac{b}{r}} = \frac{a+b}{ab}.$$

$7 \times 13 = 78$

Do you believe this? That is what has been proved (!) in the movie *In The Navy* ,a 1941 film starring the comedy team of Abbott and Costello. Three verificationsare given:

$\begin{array}{r} 13 \\ \times \quad 7 \\ \hline 21 \\ + \quad 7 \\ \hline 28 \end{array}$	$\begin{array}{r} 13 \\ 7\overline{)28} \\ 7 \\ \hline 21 \\ 21 \\ \hline 0 \end{array}$	$\begin{array}{r} 13 \\ +13 \\ +13 \\ +13 \\ +13 \\ +13 \\ +13 \\ \hline 28 \end{array}$
Explanation: $7 \times 3 = 21.7 \times 1 = 7$ Add these two.	***Explanation***:: 7 does not go into 2. So divide 8 by 7; you get 21.Now 21÷7 gives 3. Answer is thus 13.	***Explanation***: Repeated addition of 13 done; dorepeated addition of 3*s* first andadd to that repeated addition of 1*s.*

Sure, JMs will be able to comprehend the fallacy and folly in the whole process.

The Number 1089!

$$\begin{array}{r} 982 \\ -289 \\ \hline 693 \\ +396 \\ \hline 1089 \end{array}$$

Many of you are perhaps already familiar with this fun.
Write any three digit number such that the hundreds digit is at least 2 more than the units digit. Reverse the digit and subtract. Now reverse the digits and add. You always get 1089! How?

Assume as given

Hundreds	Tens	Ones
x	y	z

$$\begin{array}{lll} \textit{Original number} & = & 100x+10y+z \\ \text{Reversed} & = & \underline{100z+10y+x} \\ \textit{Subtract} & & 100(x-z)+(z-x) \\ & = & 100(x-z-1)\ +90+(10+z-x) \\ \text{Reversed} & & 100(10+z-x)+90+(x\ -z-1) \\ \textit{Add} & & 100(9) \qquad +180\ +9 \\ & = & 1089 \end{array}$$

***Learn about this shape*:** **Ellipse**

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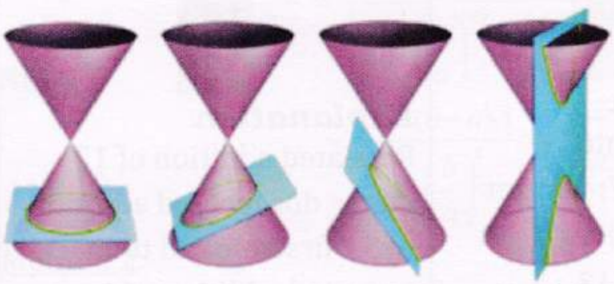
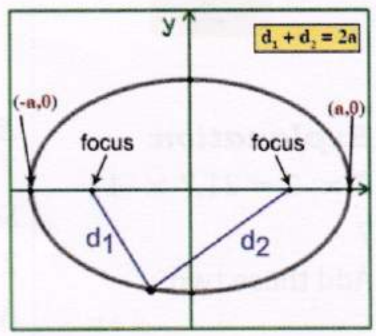
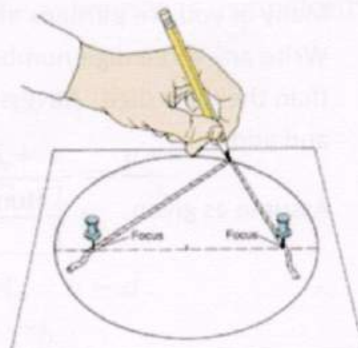
Here, you see four planes, cutting through a cone at various angles, producing the curves shown in the following diagram. The intersection of each plane with the cone forms a conic section. The angle of intersection of the plane with the cone decides the shape of the conic section.



Circle Ellipse Parabola Hyperbola

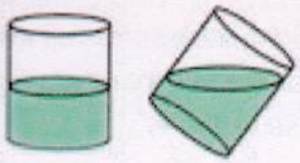


If the right circular cone is cut by a plane perpendicular to the axis of the cone, the intersection is a circle. If the plane intersects one of the pieces of the cone and its axis but is not perpendicular to the axis, the intersection will be an ellipse.

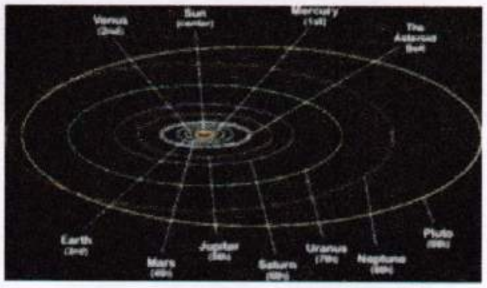


An **ellipse** sort of looks like an oval, and is the set of points whose distances from two fixed points (called foci) inside the ellipse is always the same, In the figure, $d1 + d2 = 2a$. You can use this property to draw an **ellipse.**

Stick two thumbtacks in a sheet of paper; hook a piece of string around them. Keep the string stretched with the point of your pencil; now move the pencil around to trace an **ellipse.**



Ellipses are seen all around us. Tilt a glass of water and the surface of the liquid acquires an elliptical outline. There are logos of business concerns in elliptic form. Planets orbit the sun in an elliptical pattern! In the 17th century, Johannes Kepler discovered that each planet travels around the sun in an elliptical orbit with the sun at one ofits foci.



A circle, viewed obliquely, appears elliptical.
Ellipses can also be seen when a hula hoop or tire of a car is turned askew.

An ellipse has the property that if light or a sound wave emanates from one focus, it will be reflected to the other focus. This property is used to create "whispering galleries". These are structures that allow someone who is whispering in one area to be heard clearly by someone in another area but not by anyone else. Famous examples of whispering galleries include the United States Statuary Capitol Hall and London's St. Paul's Cathedral.

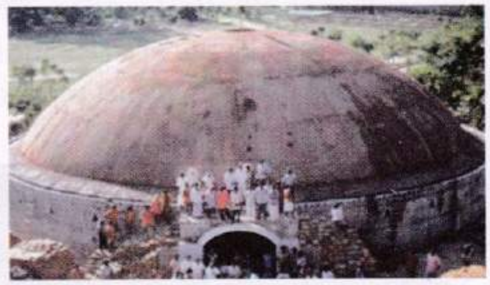
Statuary Capitol Hall (U.S)

St. Paul's Cathedral (U.K)

Some tanks are in fact elliptical (not circular) in cross section. There are a number of factors which affect the shape of a liquid carrying tank. Most important one is the stability of the vehicle on the move while taking a

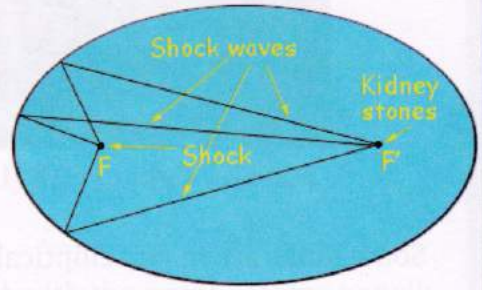
turn on the road. To have maximum safety factor, the centre of gravity of the loaded vehicle has to be as close as possible to the ground. This is achieved by adopting an elliptical shape of cross section of the tank. In fact the height of the centre of gravity will be least for a rectangular cross section. You might see these tanks transportingheating oil or gasoline on the highways.

Architecture and industry is another field that uses ellipse shapes.
Dhyanalingam Temple – Poondi, India, Segmental elliptical ø 22.16 m ,built with fired bricks Furniture, buildings, tanks, swimming pools and everywhere elliptical shapes are seen.



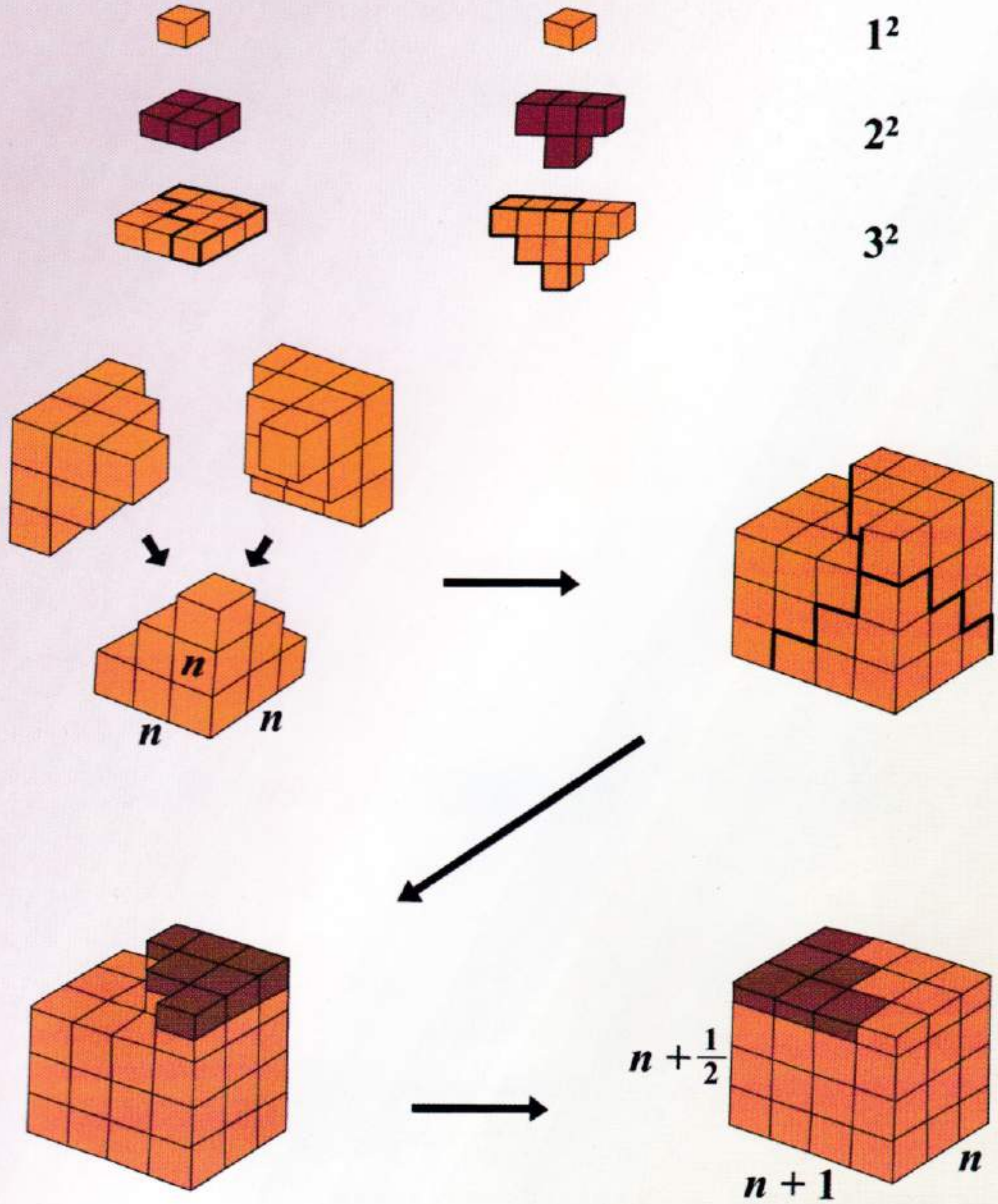
The ability of the ellipse to rebound an object starting from one focus to the other focus can be demonstrated with an elliptical billiard table. When a ball is placed at one focus and is thrust with a cue stick, it will rebound to the other focus. If the billiard table is live enough, the ball will continue passing through each focus and rebound to the other.

The reflection property of the ellipse plays an important role in medicine. *Lithotripsy*, a medical procedure for treating kidney stones, relies on this reflective principle. The patient is placed in an elliptical tank of water in such a way that the kidney stone is at one focus. High-energy shock waves, generated at the other focus, pulverize the kidney stone.



All the above details show that concepts that are discussed mathematically have real-life applications and hence should be taken up for serious study. JMs will have a good opportunity when they enter the higher secondary stage.

A Picture Story



$$\frac{1}{3}\left[n(n+1)(n+\tfrac{1}{2})\right]$$

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